Spins in a quantum 1D multi-particle environment
Munich, 2 September, 2019

Topological multicriticality of spin-orbit coupled electrons in one dimension

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supported by Vetenskapsrådet

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Some background and motivation…

Our current understanding of Quantum Phase Transitions:

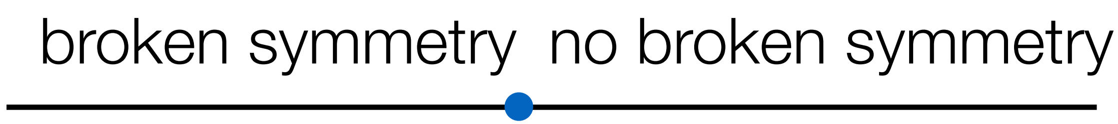
change of symmetry or topology of a ground state

Some background and motivation…

continuous finite order

Our current understanding of Quantum Phase Transitions:

change of symmetry or topology of a ground state



"Landau-Ginzburg-Wilson"

S. Sachdev, *Quantum Phase Transitions*, 2nd ed. (Cambridge, 2011)

Some background and motivation…

continuous *finite order*

Our current understanding of Quantum Phase Transitions:

change of symmetry or topology of a ground state

broken symmetry   no broken symmetry

"Landau-Ginzburg-Wilson"

broken symmetry   another broken symmetry

"deconfined quantum criticality"

T. Senthil *et al.,* Science **303**,1490 (2004)

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long-range entanglement | no long-range entanglement

"QPTs into topologically ordered phases"

X.-G. Wen, Rev. Mod. Phys. **81**, 41004 (2017)

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"QPTs into topologically ordered phases"

some topological invariant | another topological invariant

"QPTs between different symmetry-protected topological phases"

C.-K. Chiu *et al.*, Rev. Mod. Phys. **88**, 035005 (2017)

Some background and motivation…

Common feature of quantum phase transitions [QPTs] (from a gapped ground state):

QPT ⇒

nonanalytic ground-state energy

&

closing of the energy gap between the ground state and the first excited state

Some background and motivation…

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QPT ⇐ ? nonanalytic ground-state energy & closing of the energy gap between the ground state and the first excited state

Some background and motivation…

Common feature of quantum phase transitions [QPTs] (from a gapped ground state):

conjecture

QPT

nonanalytic ground-state energy

&

closing of the energy gap between the ground state and the first excited state

"Spurious" QPTs may appear across topological multicritical points.
No change of symmetry or topology!

Case study

1D electrons with modulated spin-orbit coupling

$$H = \sum_{n=1}^{N} \sum_{\substack{\alpha,\alpha' \\ =\uparrow,\downarrow}} h_{\alpha\alpha'}(n)\, c_{n,\alpha}^{\dagger}\, c_{n+1,\alpha'} + \mathrm{H.c.}$$

$$h_{\alpha\alpha'}(n) = -t\delta_{\alpha\alpha'} - i\gamma_{\mathrm{D}}\sigma^{x}_{\alpha\alpha'} - i\gamma_{\mathrm{R}}(n)\sigma^{y}_{\alpha\alpha'}$$

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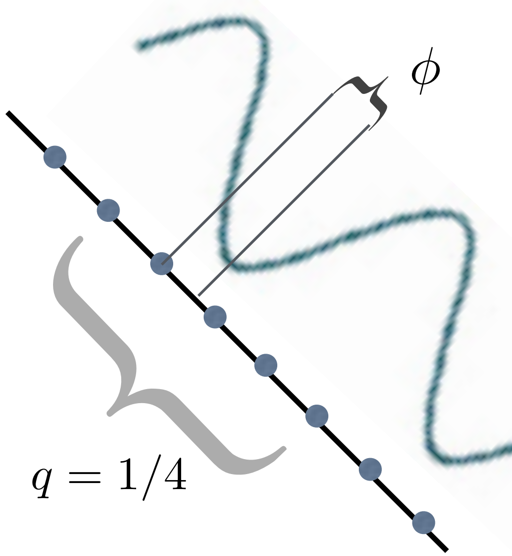

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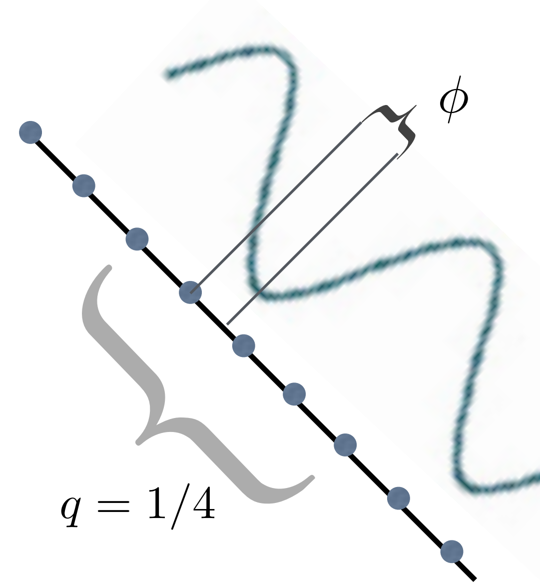


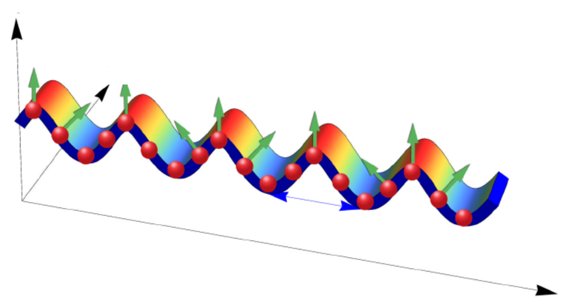
"spin-orbit generalized" *Aubry-André-Harper model* when $q \notin \mathbb{Q}$

P. G. Harper, Proc. Phys. Soc. London **A68,** 874 (1955)

S. Aubry and G. André, Ann. Isr. Phys. Soc. **3**, 133 (1980)

possible experimental realization: curved quantum wire

P. Gentile *et al.*, Phys. Rev. Lett. **115**, 256801 (2015)



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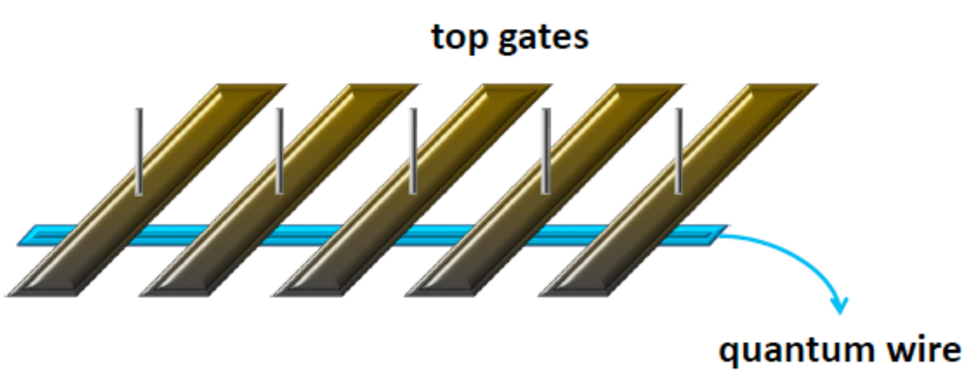
$$h_{\alpha\alpha'}(n) = -t\delta_{\alpha\alpha'} - i\gamma_{\mathrm{D}}\sigma^x_{\alpha\alpha'} - i\gamma_{\mathrm{R}}(n)\sigma^y_{\alpha\alpha'}$$

another possible realization: periodically gated quantum wire

with an added periodic chemical potential

G. I. Japaridze, H. J. & M. Malard, PRB **89**, 201403(R) (2014)



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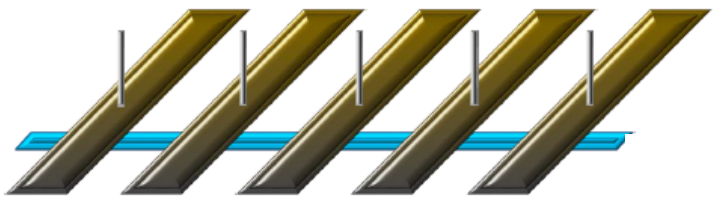
e-e interaction

$$+ \sum_{\substack{n,n' \\ \alpha,\alpha'}} V(n-n') c^\dagger_{n,\alpha} c^\dagger_{n',\alpha'} c_{n',\alpha'} c_{n,\alpha}$$



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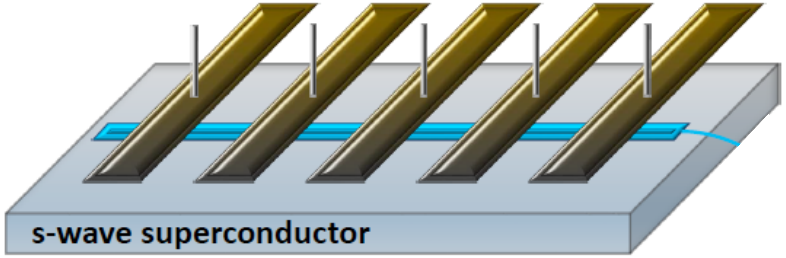

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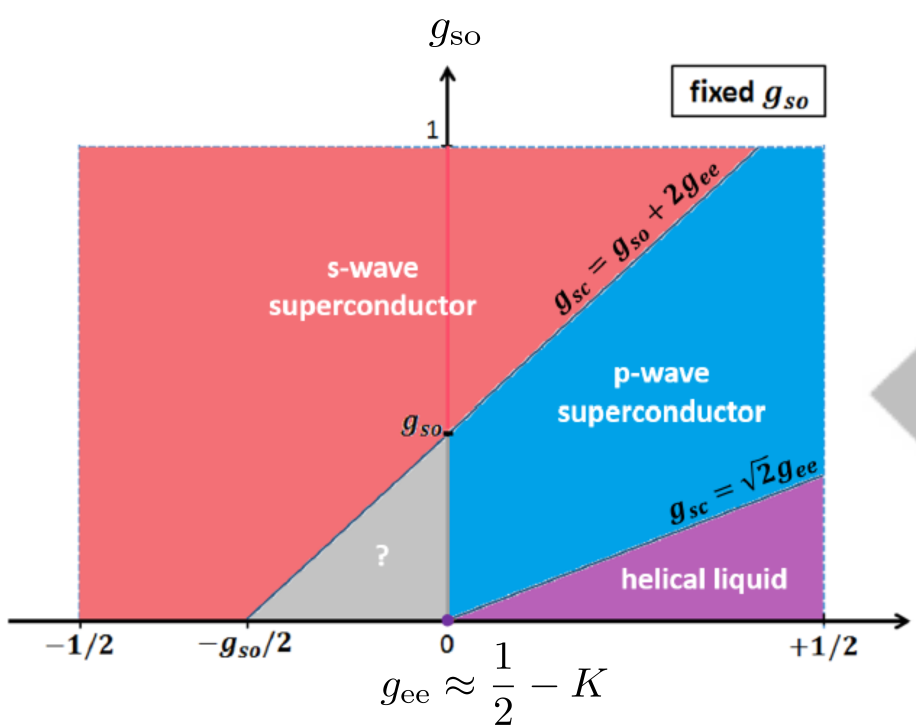

bosonization & RG

M. Malard, G. I. Japaridze & H. J., PRB **94**, 115128 (2016)

Back to the simple noninteracting model…

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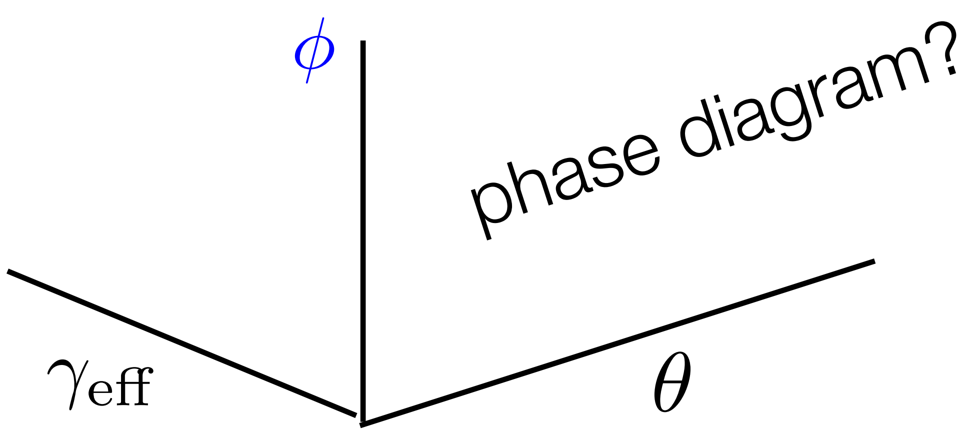
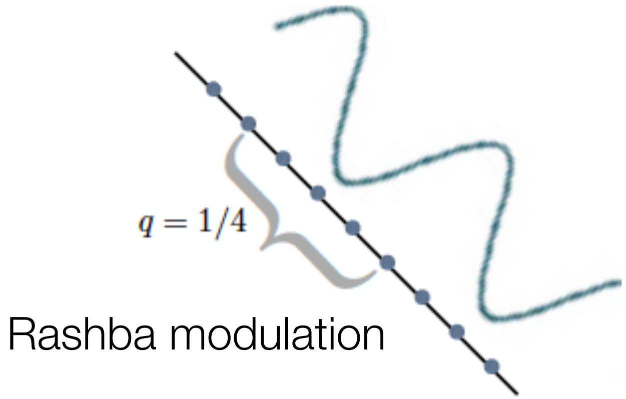


1D electrons with modulated spin-orbit coupling

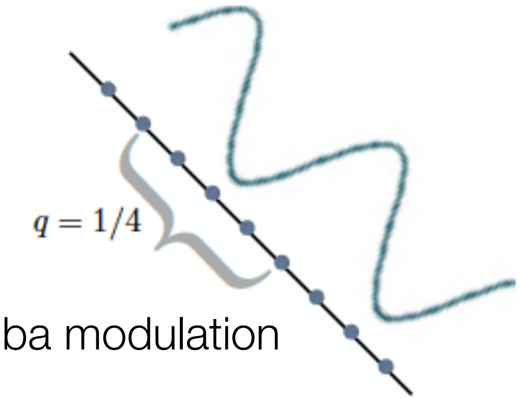


$$q = 1/4$$

Introduce an 8-component spinor in *k*-space (from Fourier transforming with respect to the unit cell position coordinates) with spin projections $\pm$ along the direction of the combined Rashba and Dresselhaus fields:

$$c_k = (c^+_{k,1}, c^-_{k,1}, c^+_{k,3}, c^-_{k,3}, c^+_{k,2}, c^-_{k,2}, c^+_{k,4}, c^-_{k,4})^T$$

1D electrons with modulated spin-orbit coupling



Rashba modulation

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$$H = \sum_k c_k^\dagger \, \mathcal{H}(k) \, c_k$$

$$= adiag(Q(k), Q^\dagger(k))$$

$$= \begin{bmatrix} A_1 & e^{-ik} A_4^* \\ A_2^* & A_3 \end{bmatrix}$$

amplitude for spin-flipping hopping between site *n* and *n*+1

amplitude for spin-conserving hopping between site *n* and *n*+1

$$A_n = \begin{bmatrix} \alpha_n^+ & \beta_n \\ \beta_n & \alpha_n^- \end{bmatrix} \quad n = 1, ..., 4$$

Symmetry class & topological invariant

chiral symmetry OK

$$\mathcal{S}\,\mathcal{H}(k)\,\mathcal{S}^{-1} \;=\; -\mathcal{H}(k) \qquad \mathcal{S} \;=\; \sigma_z \otimes \mathbb{1}_{4\times 4}$$

time-reversal symmetry OK

$$\mathcal{T}\,\mathcal{H}(k)\,\mathcal{T}^{-1} \;=\; \mathcal{H}^*(-k) \qquad \mathcal{T} \;=\; \mathbb{1}_{4\times 4} \otimes (-i\sigma_y),\; \mathcal{T}^2 = -1$$

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enforcing all three symmetries

symmetry class CII

A. P. Schnyder *et al*., PRB **87**, 195125 (2008)

topological invariant:

winding number W $\in 2\mathbb{Z}$

Topological phase diagram from computing *W*

$$W = -\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{d\varphi}{dk}\,dk, \quad \det[Q(k)] = R(k)e^{i\varphi(k)}$$

J. K. Asbóth *et al.,* Lecture Notes in Physics, 919 (2016)

Topological phase diagram

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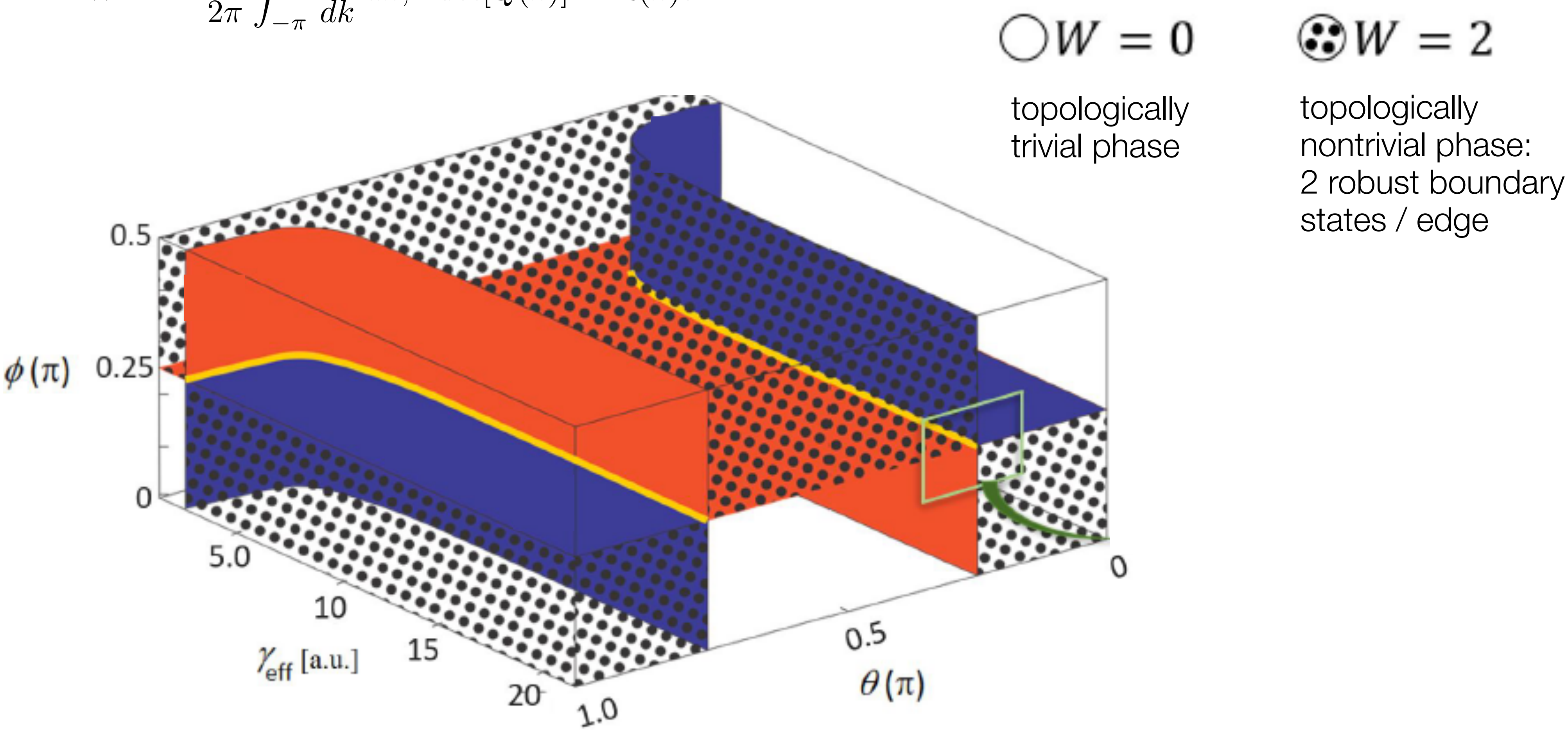

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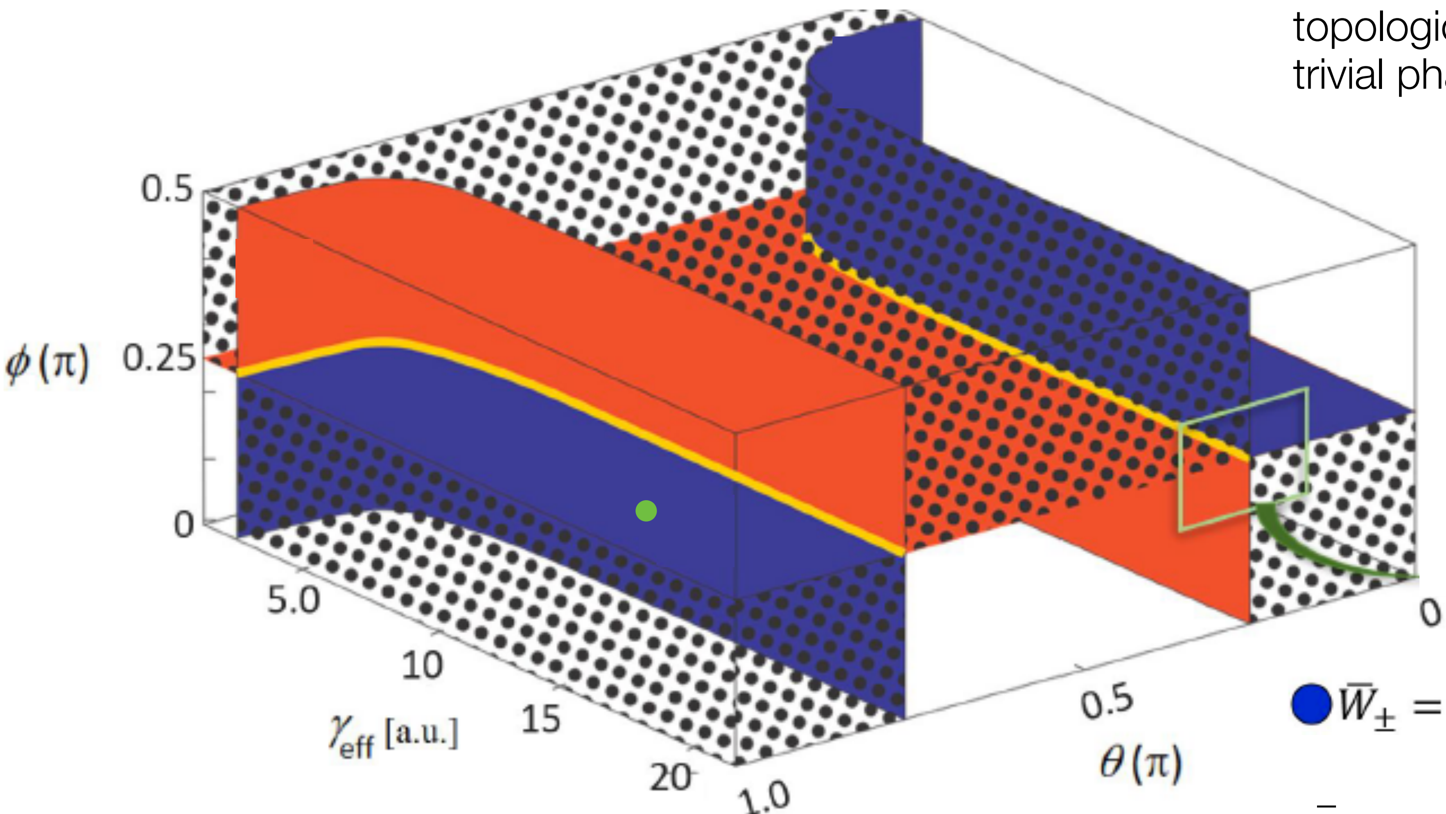

$W = 0$: topologically trivial phase

$W = 2$: topologically nontrivial phase: 2 robust boundary states / edge

$\bar{W}_\pm = -1$ $\bar{W}_\pm = 0$ $\bar{W}_\pm = 1$

$$\bar{W}_\pm = -\frac{1}{2\pi}\int_{C_\pm}\frac{d\varphi}{dk}\,dk$$

closed curve around gap-closing point $(\bar{\gamma}_{\text{eff}}, \bar{\theta}, \bar{\phi}, k_\pm)$ in parameter-momentum space

L. Li and S. Chen, PRB **92**, 085118 (2015)

Topological phase diagram

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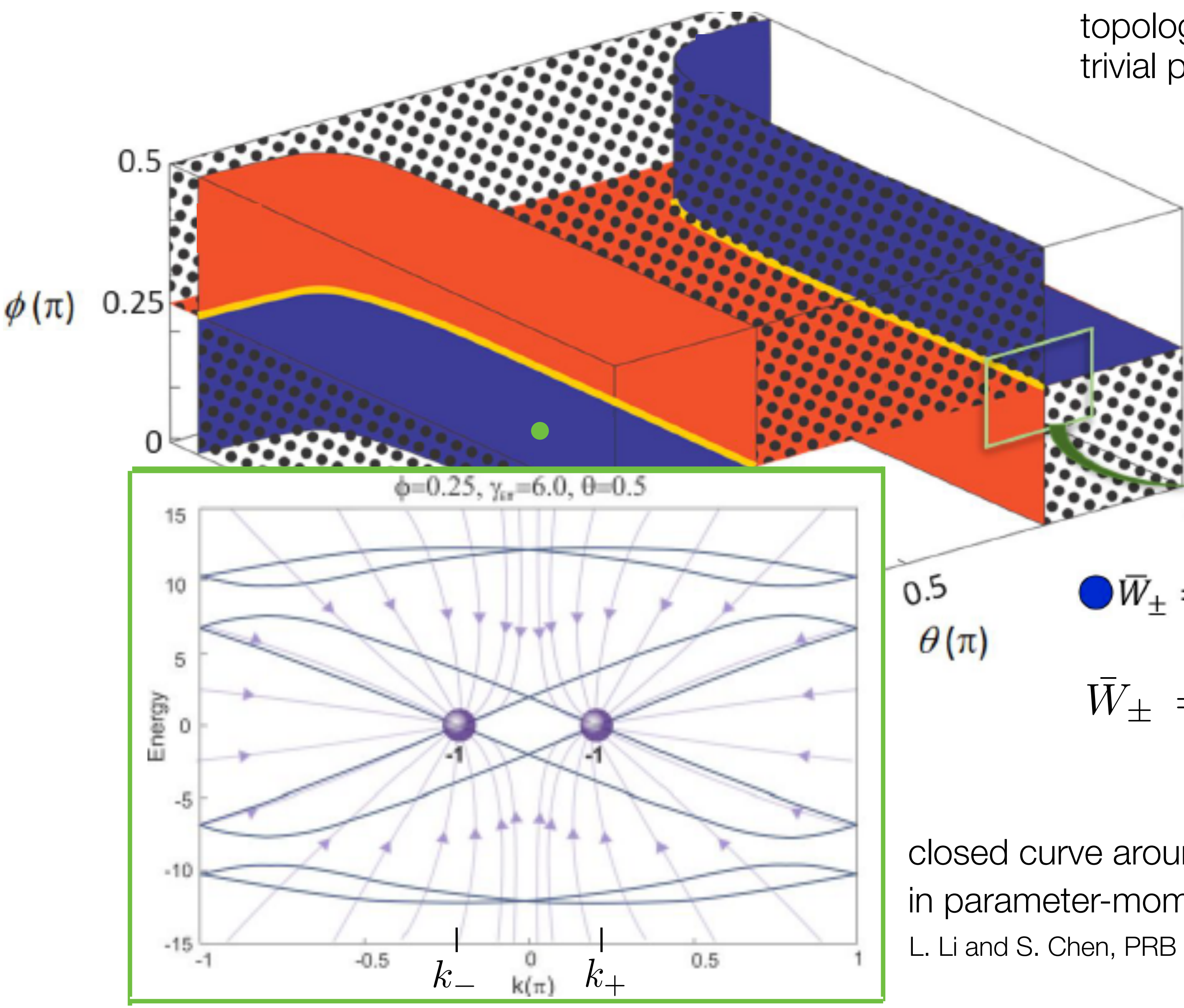

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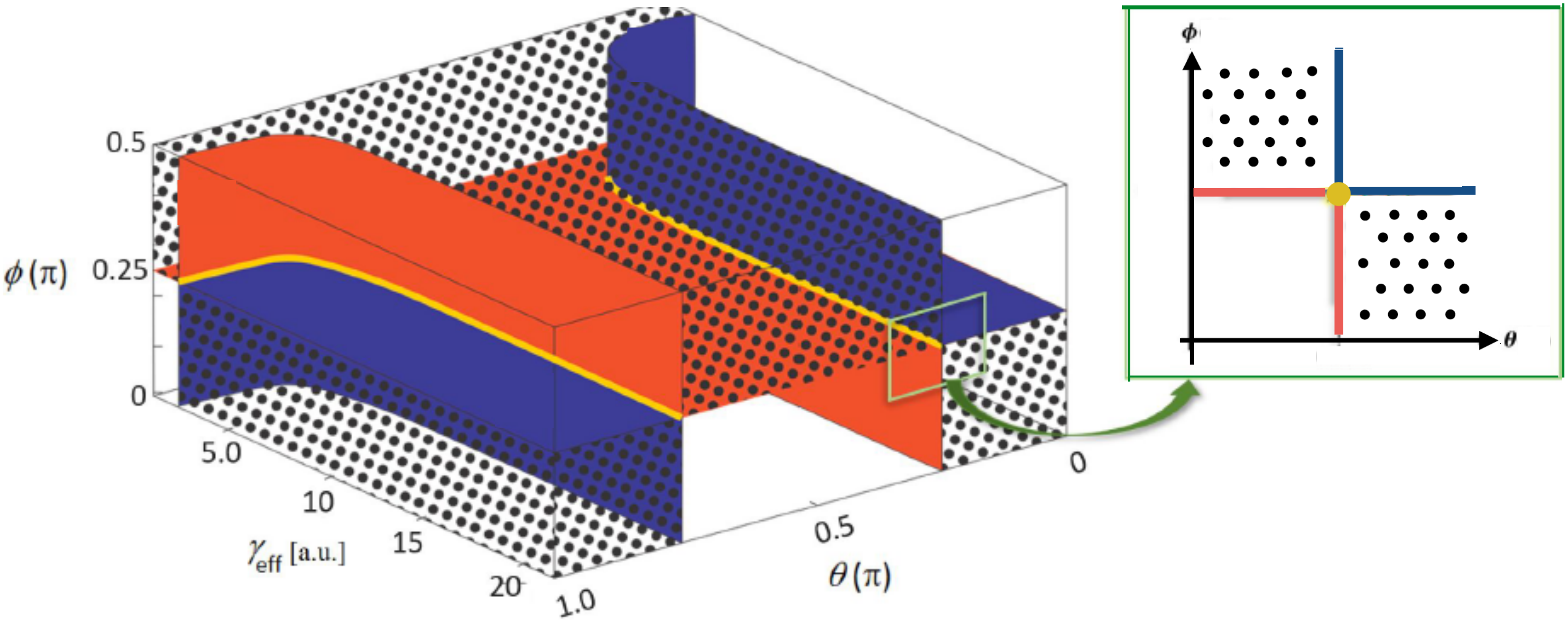

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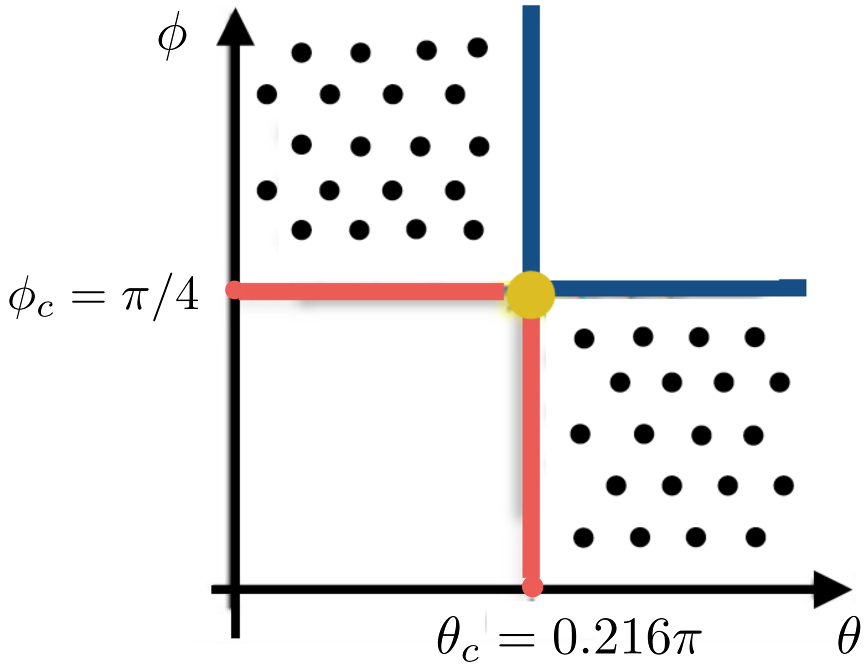

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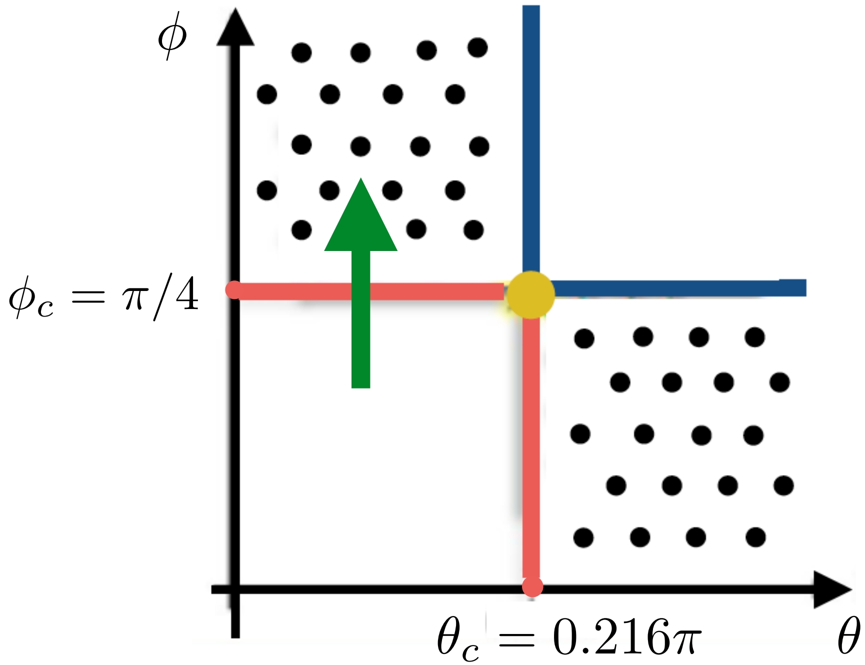

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topologically
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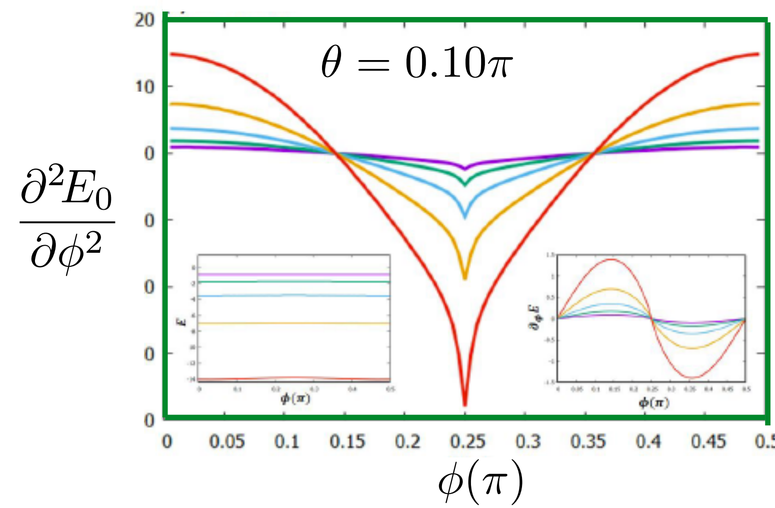
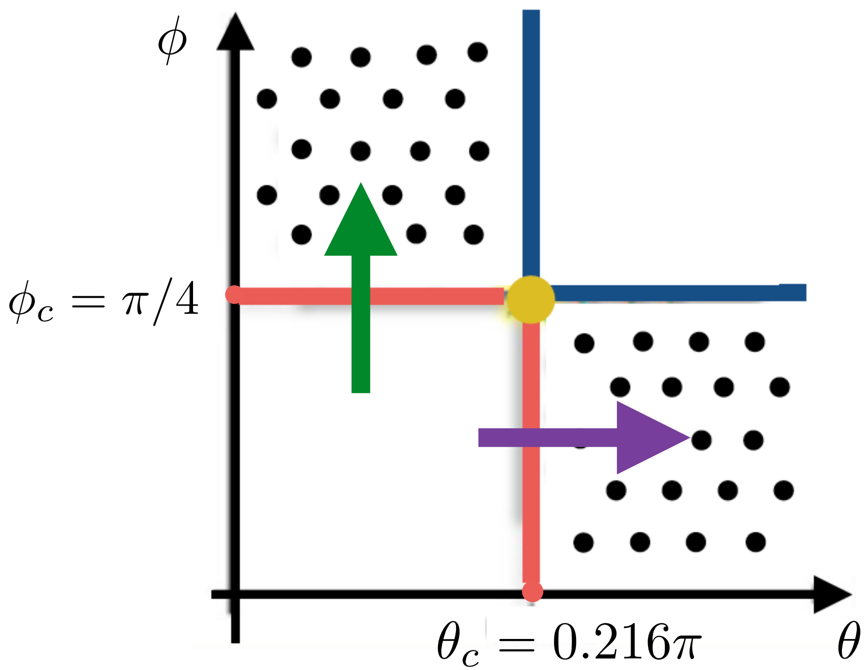
2nd order QPT

expected!

S. N. Kempkes *et al.*,
Sci. Rep. **6**, 38530 (2016)



Topological phase diagram



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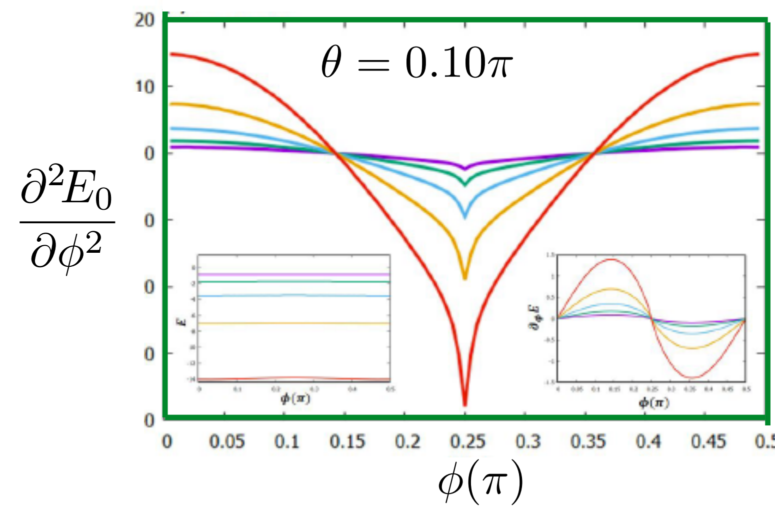
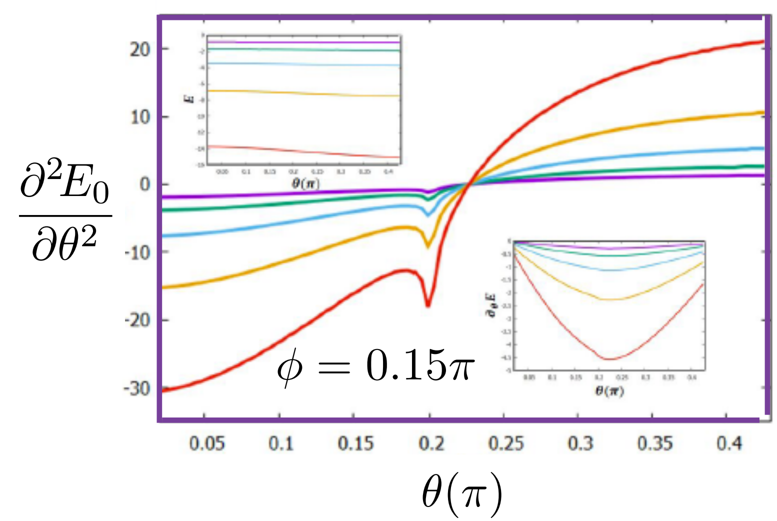
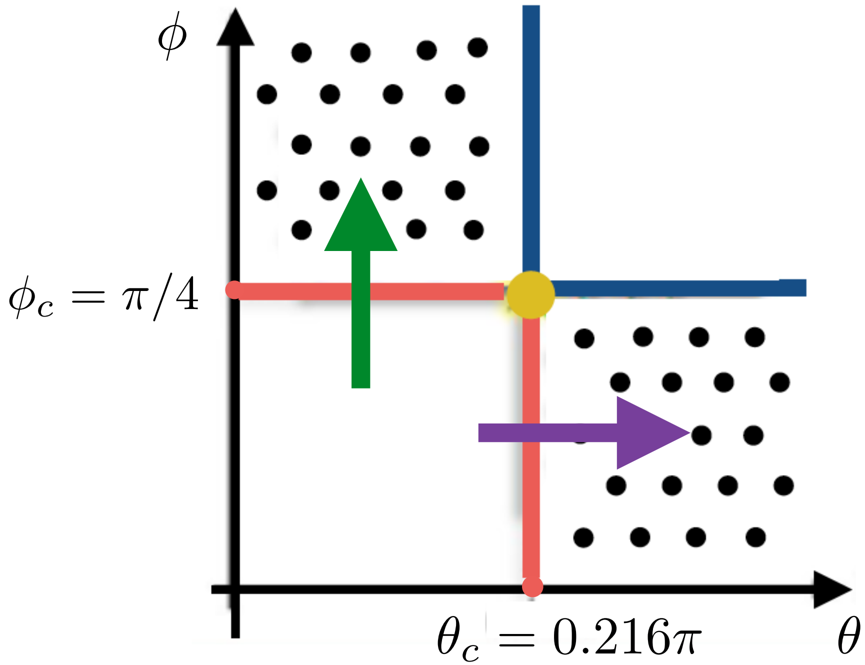
2nd order QPTs





Topological phase diagram



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topologically
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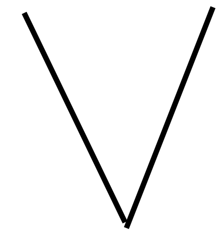
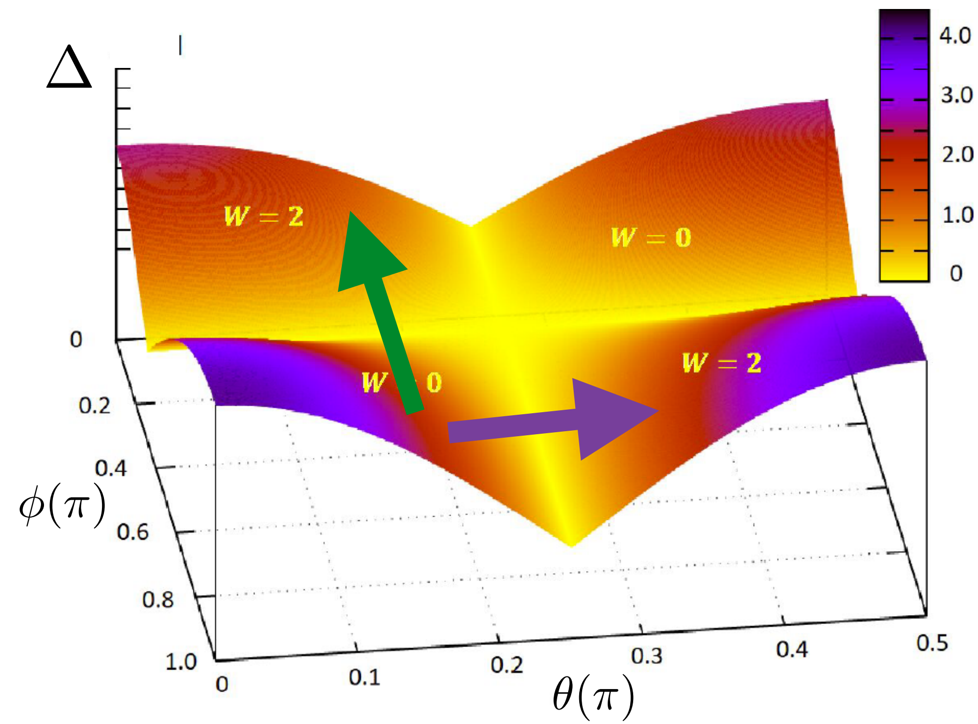
topologically
nontrivial phase:
2 robust boundary
states / edge

conical gap profiles

also expected!





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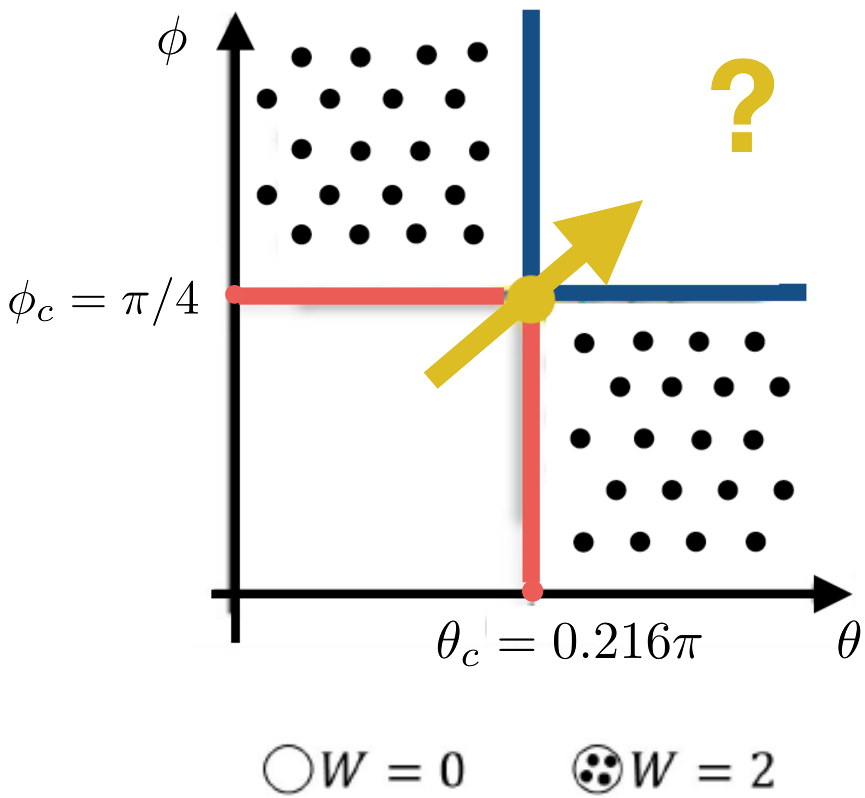

topologically
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topologically
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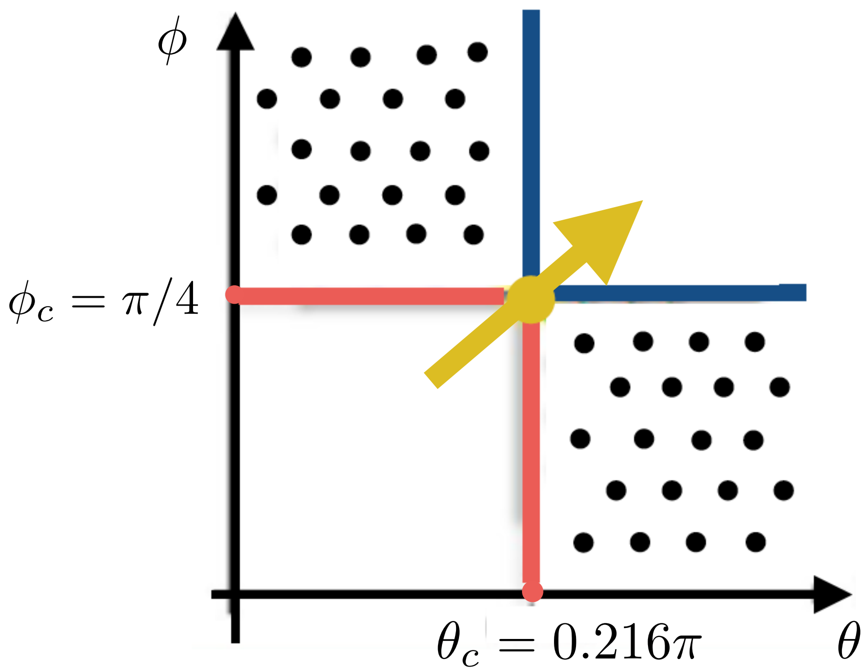

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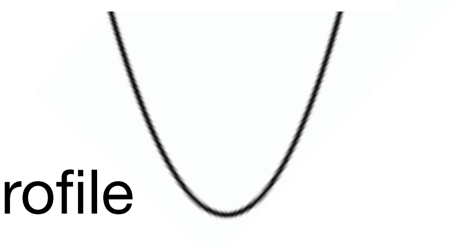
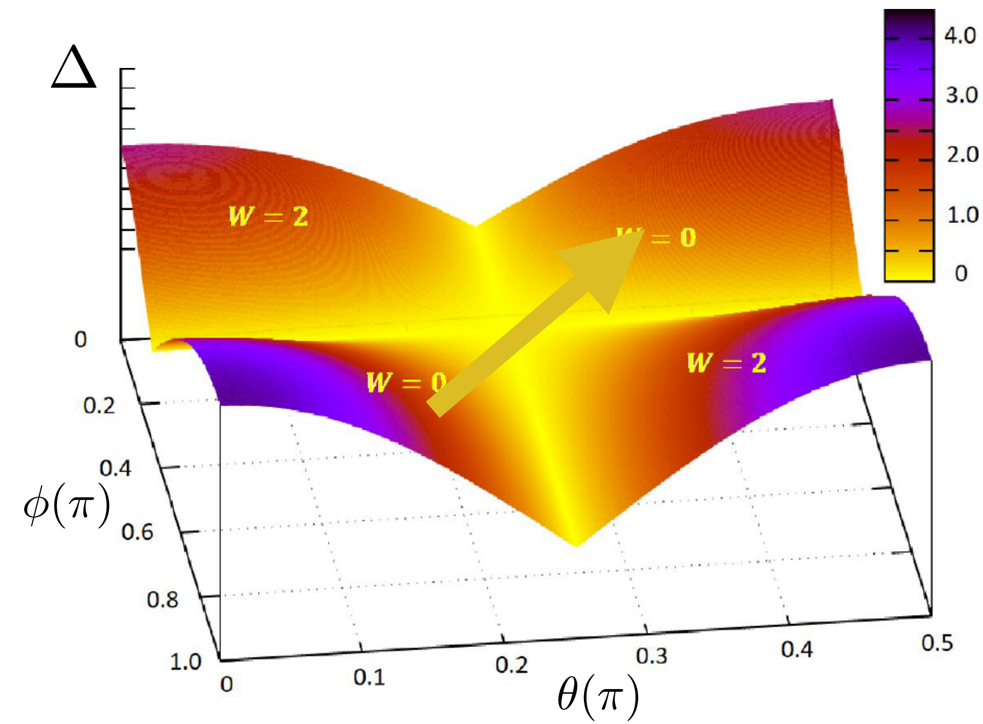
topologically nontrivial phase: 2 robust boundary states / edge

parabolic gap profile





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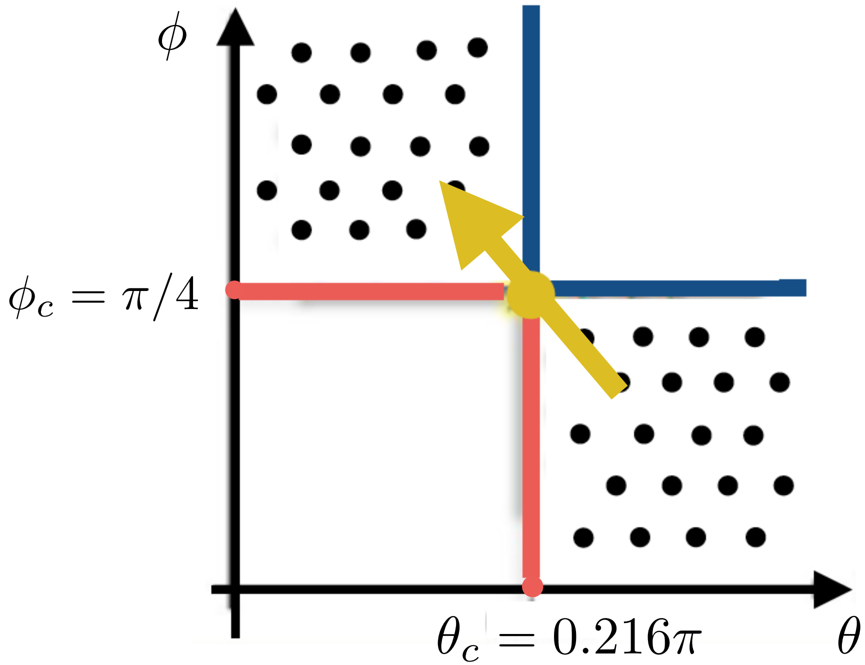

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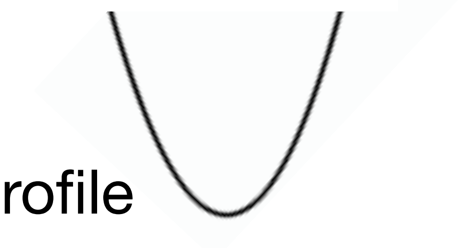
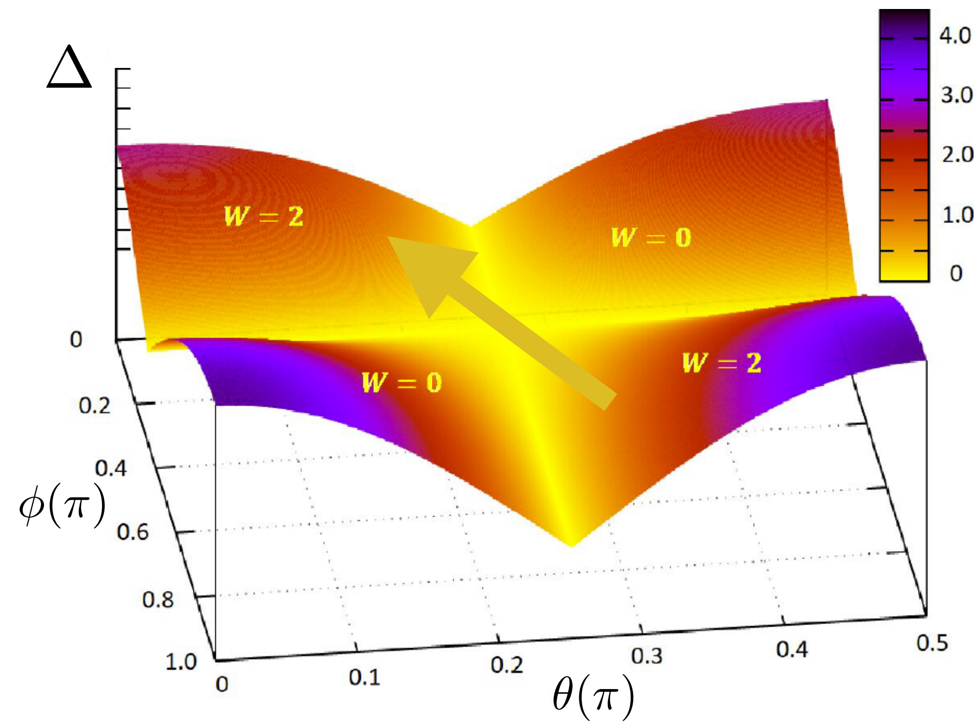
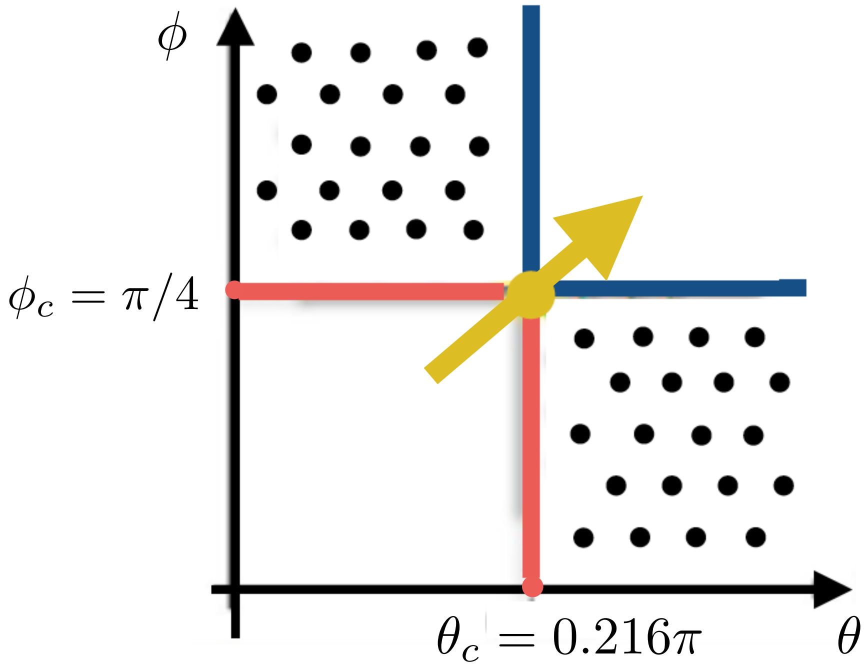




Topological phase diagram



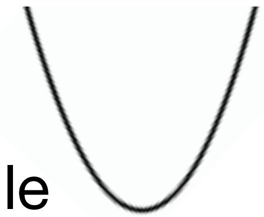
○ $W = 0$

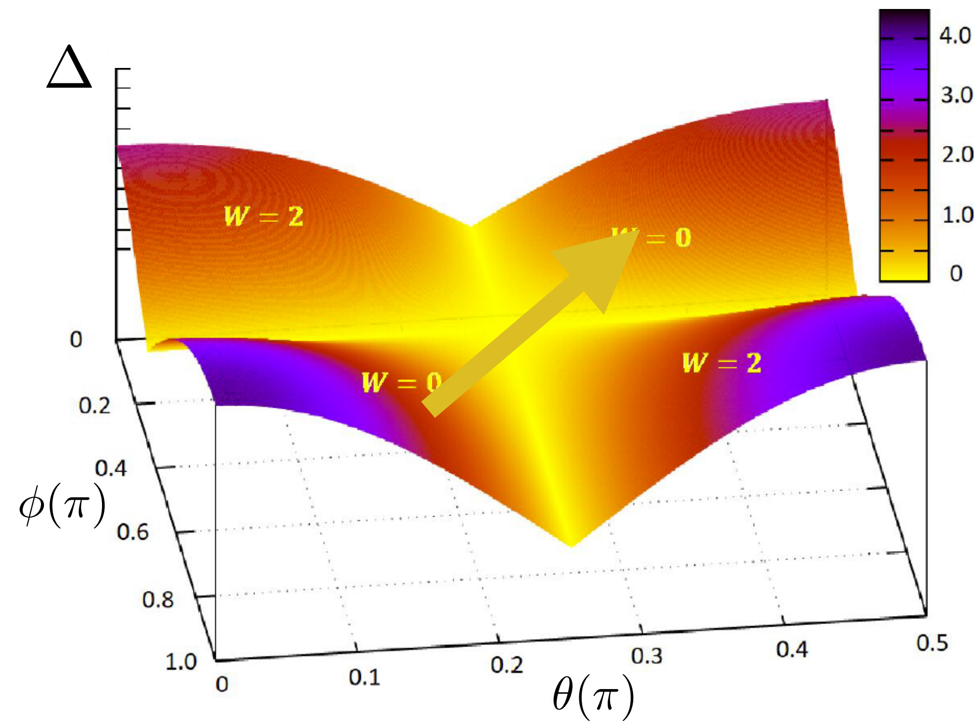
topologically
trivial phase

⦿ $W = 2$

topologically
nontrivial phase:
2 robust boundary
states / edge

parabolic gap profile





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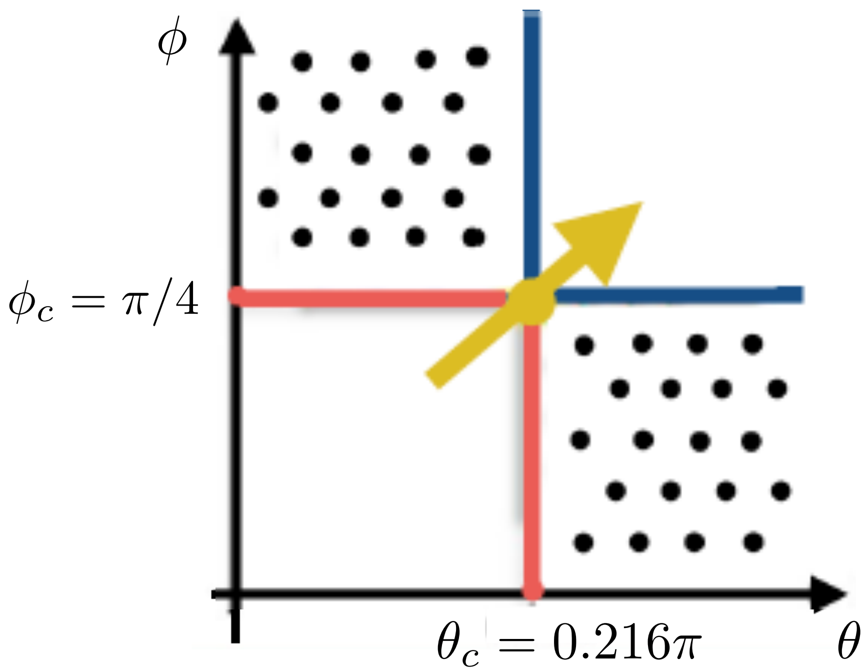

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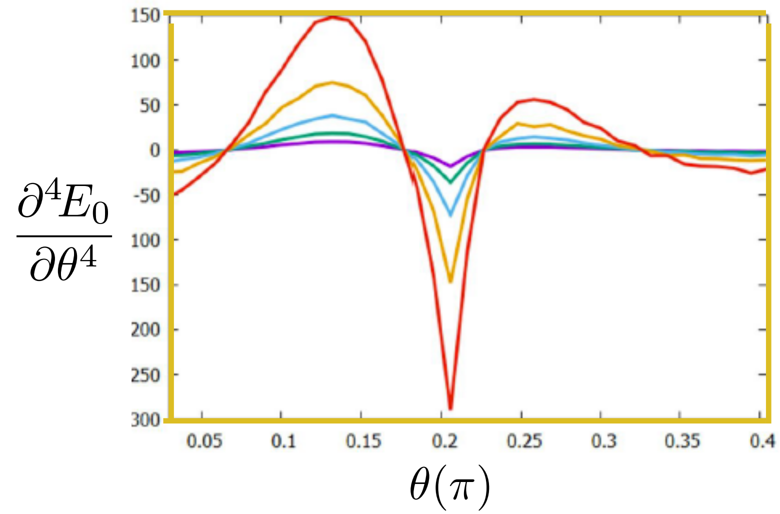
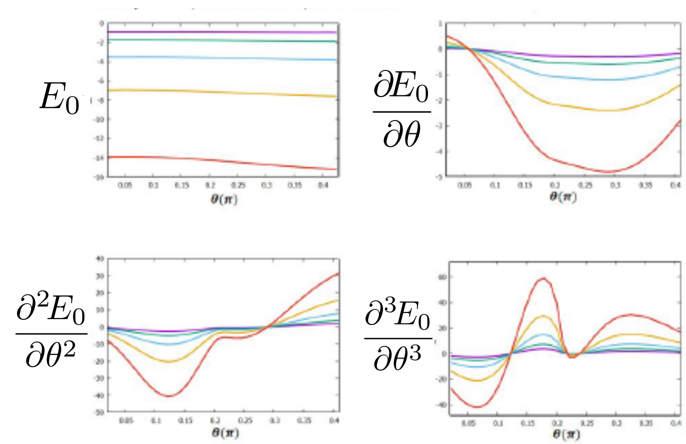
topologically nontrivial phase: 2 robust boundary states / edge

4th order nonanalyticity





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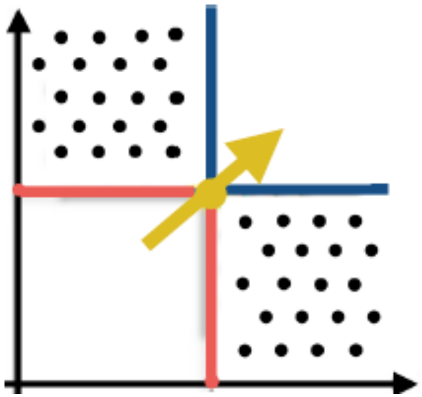
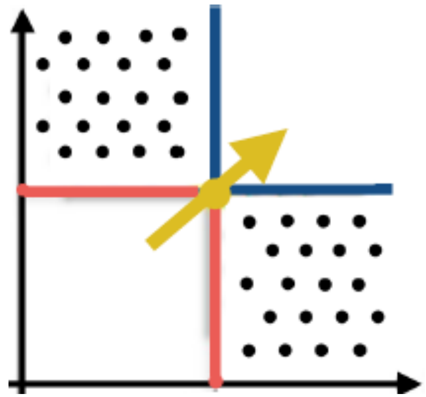


Spontaneous symmetry breaking in
one of the two *W=0 (W=2)* regions?

No. The ground state of a band insulator
(with periodic boundary conditions) is unique.

Topological phase diagram



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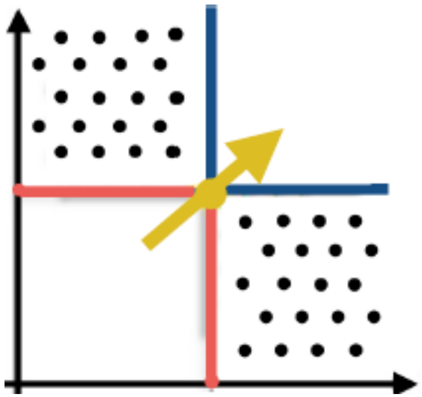

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Do the two *W=0 (W=2)* regions in fact represent distinct topological phases, identifiable by going beyond the Altland-Zirnbauer ”ten-fold” way?

Beyond the ”tenfold way”…

case in point

adding space group symmetries to the symmetries of the tenfold way:

topological crystalline insulators

L. Fu, PRL **106**, 106802 (2011)

1D: inversion, translation, mirror symmetry

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L. Fu, PRL **106**, 106802 (2011)

1D: inversion, translation, mirror symmetry

+ time-reversal symmetry

trivial 1D *AII* phase splits into trivial $(\nu = 0)$ and topologically nontrivial $(\nu = 1)$ phases

A. Lau *et al.*, PRB **94**, 165164 (2016)

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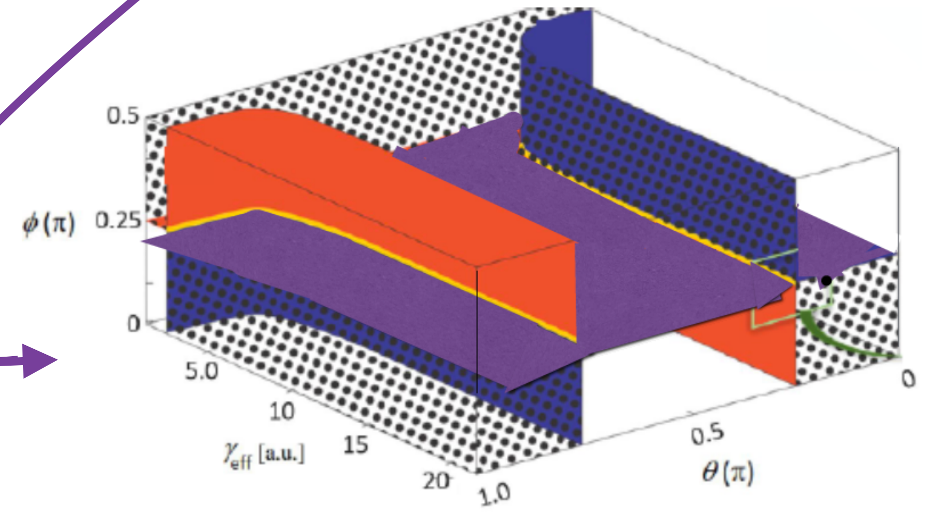

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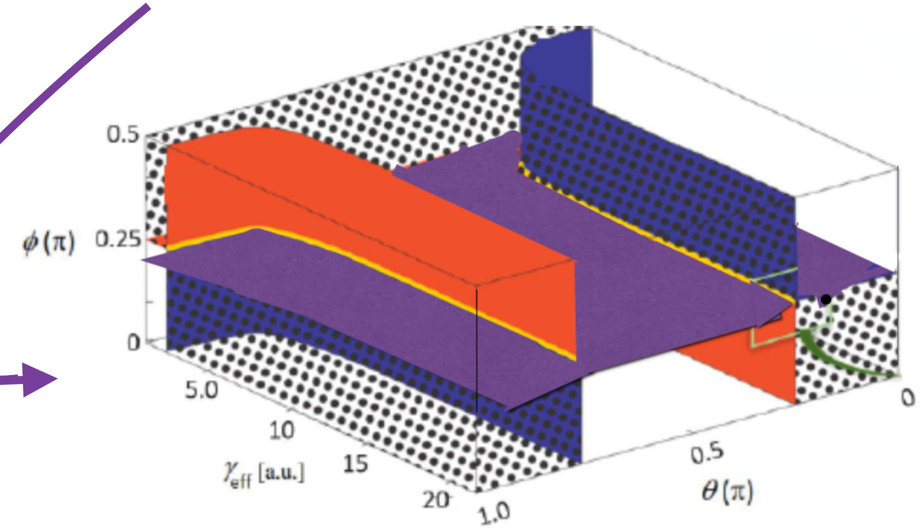
The mirror symmetry, together with time-reversal and chiral symmetry, *enforces* pairs of nodal points in the band structure, *without the presence of a nonsymmorphic symmetry!*

M. Malard, P. de Brito, S. Östlund, and H. J., PRB **98**, 165127 (2018)



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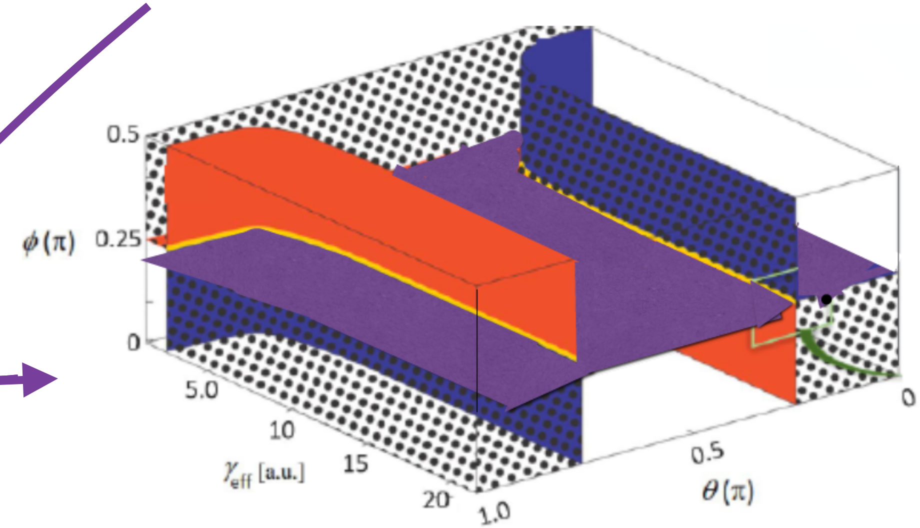

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Where to
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Alternatively… (conjecture):

A many-body ground state in the proximity to a topological QPT may occasionally develop a nonanalyticity with a simultaneous closing of the gap to the first excited level, *without undergoing a QPT*.

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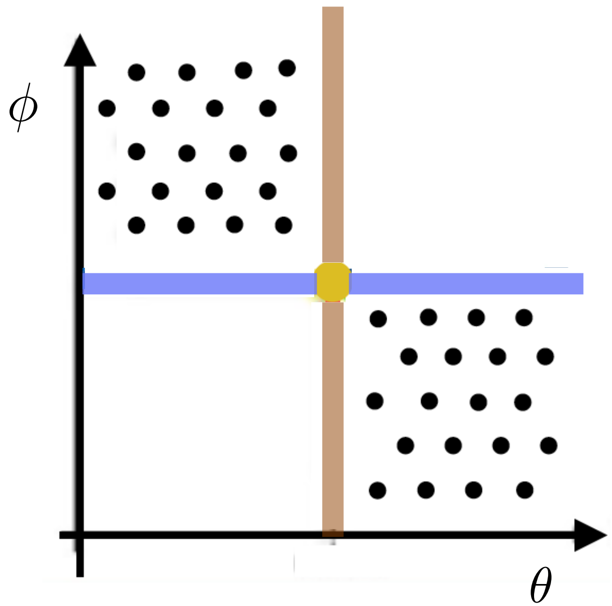

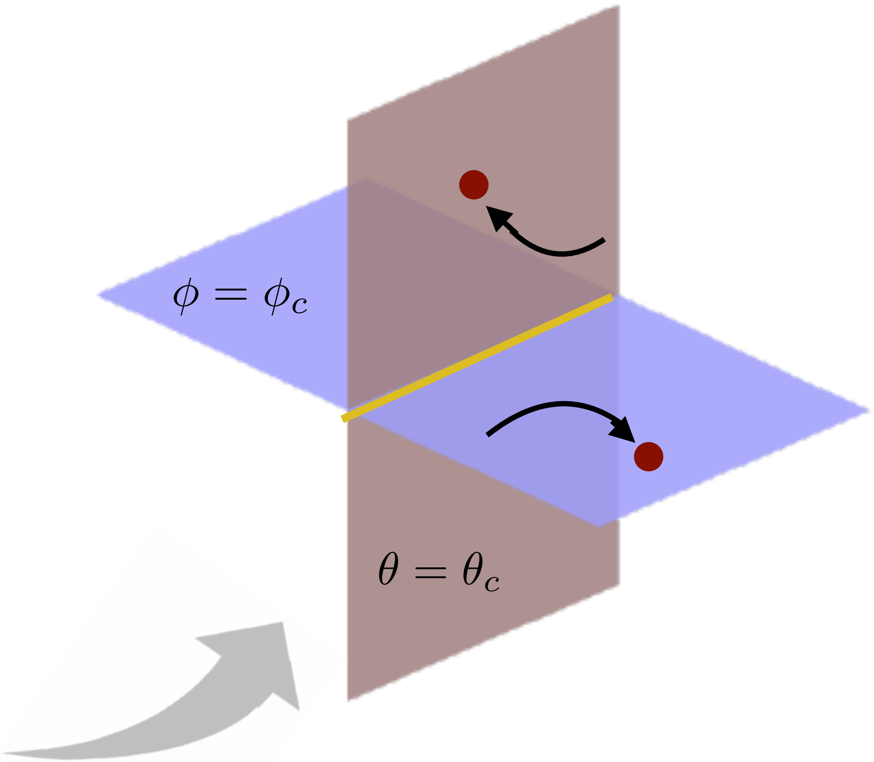

Renormalization Group picture
Intersection of RG critical surfaces at the multicritical point

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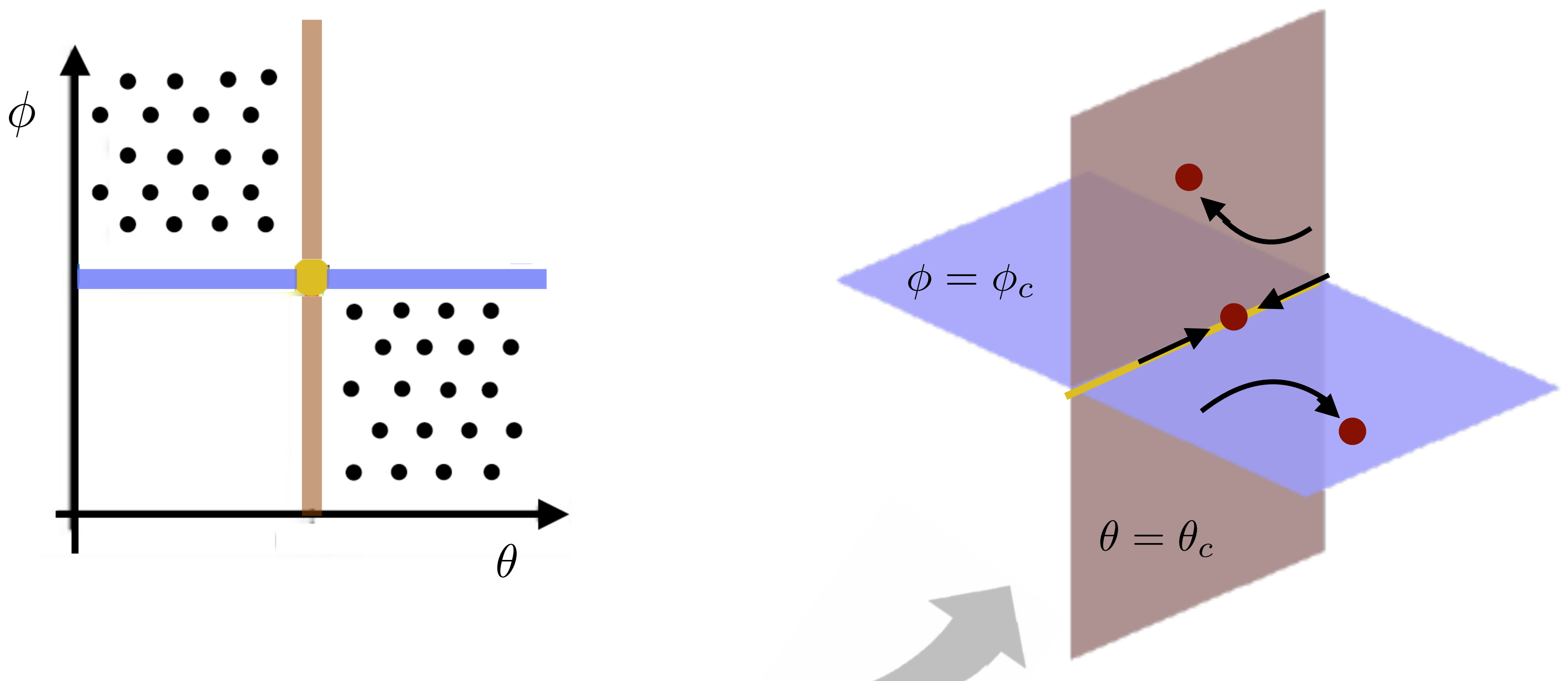

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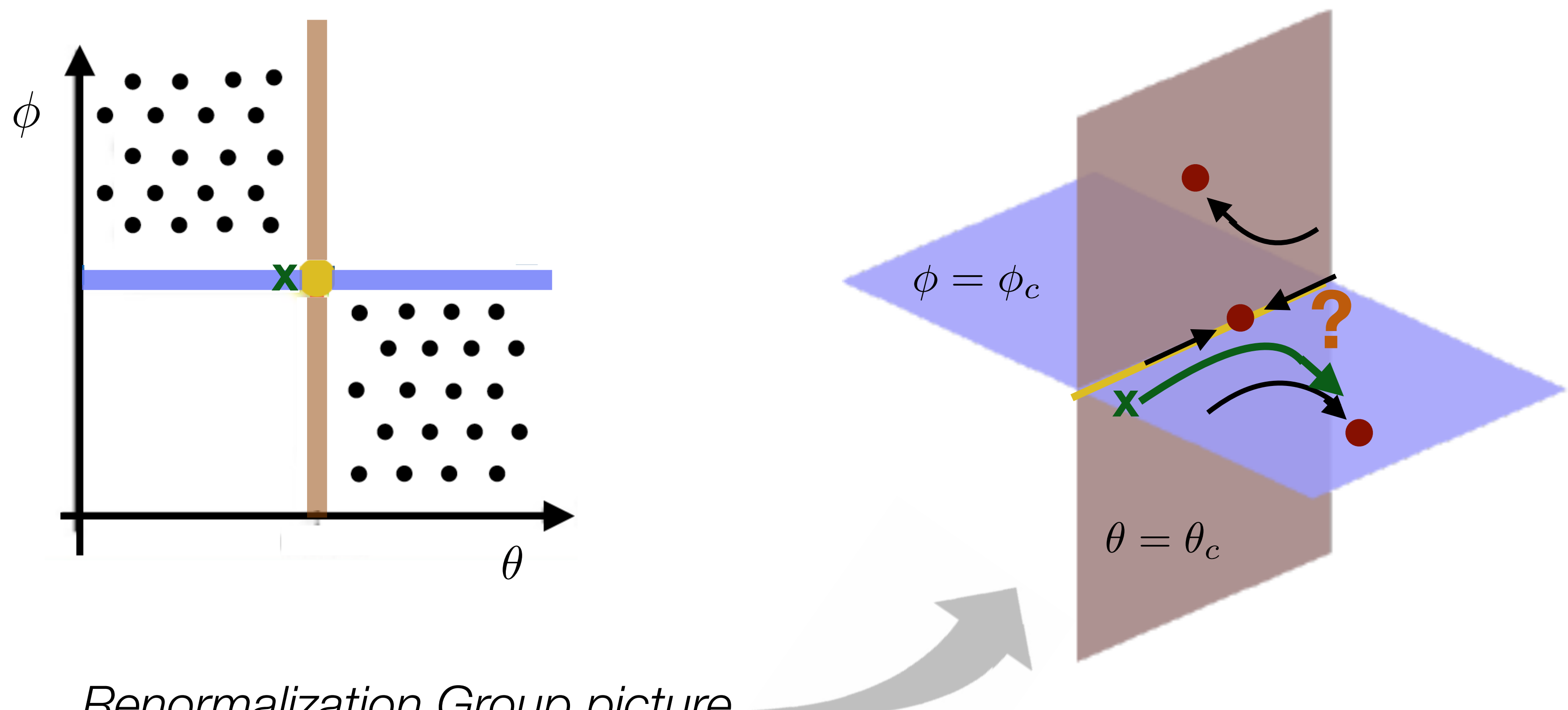

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No numerical support for the expected strong crossover behavior

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Renormalization Group picture for topological QPTs?

… in the making!

E. P. L. van Nieuwenburg *et al.*, PRB **97**, 155151 (2018)
W. Chen and A. P. Schnyder, New. J. Phys. **21**, 073003 (2019)
M. A. Continentino *et al.*, arXiv:1903.00758

… application to topological multicriticality

M. Malard, P. E. de Brito, H. J, and W. Chen, *in progress*

Summary

A 1D band insulator in symmetry class CII – with electrons subject to a spatially modulated spin-orbit coupling – has been found to support multicritical lines at which the gap closes and the ground state energy becomes nonanalytical, but *with no apparent phase transition occurring.*

How to properly understand this anomaly remains an open problem…

M. Malard, D. Brandao, P. E. de Brito, H. J., *soon to appear on the arXiv*

related work

G. I. Japaridze, H. J., M. Malard, PRB **89**, 201403(R) (2014)

M. Malard, G. I. Japaridze & H. J., PRB **94**, 115128 (2016)

M. Malard, P. E. de Brito, S. Östlund, and H. J., PRB **98**, 165127 (2018)